APPLICATION OF QUEUEING THORY IN A HOSPITAL PHARMACY UNIT

Prof. J.K. Sharma
Director, GL Bajaj Institute of Management and Research,
GREATER NOIDA (UP)

ABSTRACT

The objective of this study was to analyse various design alternatives in determining the manpower requirement needed to run a hospital's pharmacy unit efficiently. This study enable the hospital administration in understanding and effectively using the manpower available in reducing the waiting time of prescription orders under different conditions. Three different operating procedures were evaluated in order to give a complete analysis of the prescription order process taking place in the pharmacy unit. Multiple server queueing with no priorities, a priority discipline queueing model without and with pre-emptive_service has been used in this paper A numerical example has been added to demonstrate the application of queueing theory.

INTRODUCTION

In health care organizations, the practice of pharmacy is rapidly progressing toward a clinical role for the pharmacist to ensure safe, rational and cost effective drug. Pharmacists are expected to assume the primary managerial responsibilities for operating the hospital pharmacy. However, in a cost containment environment, this added responsibility is subject to the critical constraint that no additional personnel will be assigned to assist the pharmacist. Several health care organizations have been successful at accomplishing this change in the practice of pharmacy. These organizations have better managed their resources through the increased utilization of technology and technical personnel. A better management of resources has allowed an adequate time for pharmacists to assume clinical responsibilities. In the current drug delivery system, the pharmacists is involved after the physician orders medication for the patient. There may be a considerable long time between the time the physician writes the order and the time pharmacist processes that order. In the event of an error or uncertainty, contacting the physician may be difficult.

Many analytical decision medels have been used to solve personnel shift scheduling problems. et al have studied the multi objective nurse scheduling model.Bicket et al.[2] have studied the purchasing and inventory control in hospital pharmacy department. Lehnay et al.[4] Mehrez et al.[5] and Sharma et al.[6] have studied the multi objective model and their areas of application adopted. Woller [7] has studied the role of pharmacists in the pharmacy department. The present paper was to analyse various design alternatives in determining the manpower requirements needed to run a hospital pharmacy unit by using queueing theory.

DATA OF THE PROBLEM

This study was carried out in the Government Hospital The primary responsibility of the pharmacy unit is to fill prescriptions. During the day, prescription orders are delivered either by messenger or by service personnel at the pharmacy's service window. They arrive both as individual orders and in bulk. The pharmacist must perform a patient profile on each prescription and up date their records. When filled, the pharmacist checks afterwards to see that it was done accurately.

Arriving orders are usually classified into two categories and processed according to a priority system. The most urgent or slat orders are processed immediately so that service on non-slat prescriptions in progress is pre-empted. All other arrivals are considered regular orders and include new and refill prescription for non-emergency units, auxiliary hospital units and floor supplies. Because of. probabilistic elements in the pharmacy's operation, queues tend to form. Prescription orders arrive at random and the time required to fill an order is random. When no pharmacist is available for service, orders must wait in line. The working of pharmacy unit was observed for one week duration in order to determine the nature of the queue, arrivals and services. Throughout this time, the data were collected during the busiest part of the day.

QUEUE DISCIPLINE

A single line forms for all prescription orders wait for service. There is always more than one pharmacist working at any time. The sequence in which prescriptions are filled is based on a priority system. Two priority classes exist: (a) with highest priority orders and (b) regular orders, including other new prescriptions, refills and floor supplies, receiving the lowest priority. In addition, service is pre-emptive so that service on an order is interrupted if a higher priority order enters the queueing system. The low priority regular order resumes service from the point at which it was pre-empted when there are no more statorders waiting to be processed.

ARRIVAL PATTERN

Prescriptions arrive throughout the day, being delivered by messenger or service personnel at the pharmacy window. Though some arrivals are make up of several prescription orders in bulk, they are treated as individual orders arriving at the same time. Arrivals were measured separately for the two priority classes when data were collected over consecutive 20 minute intervals. The mean arrival rate λ_1 and λ_2 of highest priority orders and regular orders are found to be 0.092 and 0.27 per minute, respectively.

SERVICE PATTERN

Data were obtained for the time required to service prescription orders for each of the two priority classes. In that time, the pharmacist fills the prescription and up dates his records. Each order processed during a one month period was recorded. It was found that the average service time for highest priority orders ($1/\mu_1$) and regular orders ($1/\mu_2$) were 1.62 and 5.71 minutes, respectively. Both types of orders follow general service distribution.

DEVELOPMENT OF QUEUING MODELS

In order to apply analytical formulae to a priority discipline queueing model it is necessary to make the assumption that the mean service time is the same for all priority classes and form an exponential distribution. Since single service time distribution lies somewhere between $1/\mu_1$ and $1/\mu_2$,therefore,

$$1/\mu = (\lambda_1 / \lambda) /\mu_1 + (\lambda_2 / \lambda)/\mu_2 \qquad (1)$$

where $\lambda = \lambda_1 + \lambda_2$. In this problem $\lambda = 0.362$. Then the mean service time, $1/\mu$ is 4.67 and is assumed to be the parameter of all exponential distribution. Since the system utilization rate, $\lambda/s\mu$ is greater than 1, more than one pharmacist is needed. To study the system under steady-state conditions, it is necessary that $\lambda/s\mu < 1$, where s is the number of servers.

Three different operating procedures are designed and evaluated in order to give a complete analysis of the prescription order process taking place at the pharmacy unit. For each procedure the expected waiting time of an order, Lq is the charcteristic of interest as a function of the number of pharmacists.

DESIGN I : MULTIPLE SERVER QUEUEING MODEL WITH NO PRIORITIES

In this queueing model no priority is given to orders and there is a single arrival rate and service time for all orders. Assuming that the arrival rate is Possion and service time is exponential, we have the following standard results :

For Design I : $\lambda = \lambda_1+\lambda_2 = 0.362$ and $\dfrac{1}{\mu} = 4.67$

(i) Probability that no orders are waiting is

$$P_0 = \left(\sum_{j=0}^{s-1} \frac{(\lambda/\mu)^j}{j!} + \frac{(\lambda/\mu)^s}{s!\,(1-\lambda/s\mu)} \right)^{-1}$$

(ii) Probability that as servers are busy is :

$$P\text{ (busy period)} = \frac{(\lambda/\mu)^s P_0}{s!\,(1-\lambda/s\mu)}$$

(iii) Expected waiting time of an order is :

$$Lq = \frac{P(\text{busy period})}{(s\mu-\lambda)} \qquad (2)$$

DESIGN II : PRIORITY DISCIPLINE QUEUEING MODEL

In this queueing model we considered to priority classes where the arrival process for each class is Possion, service is non-pre-emptive and service times are assumed exponential for each priority class with the same mean service time. In this model priorities are: (i) high priority orders and (ii) regular orders

For, $\lambda_1 = 0.092$; $\lambda_2 = 0.27$; $\lambda = 0.362$ and $1/\mu = 4.67$, we define

(iv) $\sigma_m = \Sigma\, \lambda_j / \mu$; for all m priority classes.

where $\sigma_0 = 0$; $\sigma_1 = 0.429$ and $\sigma_2 = 1.619$

The expected waiting time for orders class m is:

(v) $L_{qk} = s/\mu \;\text{x}\; P(\text{busy period}) / (s - \sigma_{m-1})\,(s - \sigma_m)$

DESIGN III : PRIORITY DISCIPLINE QUEUEING MODEL WITH PRE-EMPTIVE SERVICE

This model is same as model II but allows the pre-emptive service. In Design III, the waiting time for priority class I customers are not effected by the presence of orders in lower priority classes when pre-emptive service *is* introduced. Therefore, L_{q1} is found using E_{qn} (2) where $1/\mu = 4.67$ and $\lambda=\lambda_1 = 0.092$. Now let $L_{q1,2}$ be the expected waiting time of a random arrival in either class 1 or 2, then the probability is $\lambda/(\lambda_1 + \lambda_2) = 0.254$ that this arrival is in class 1, and $\lambda_2 /(\lambda_1 + \lambda_2) = 0.746$ that it is in class 2. Therefore, $L_{q1,2} = 0.254$, $L_{q1} + 0.746$ L L_{q2} and $L_{q1,2}$ can be found using E_{qn} (2) with $1/\mu = 4.6\ 7$ and $\lambda = 0.362$. Thus

$$Lq_2 = \left[(\lambda_1 + \lambda_2))/\lambda_2)\right] L_{q1,2} - (\lambda_1/\lambda_2)\, L_{ql}$$

$$= 1.341 L_{q1,2} - 0.341\, L_{ql}$$

RESULTS AND DISCUSSION

For each of the three design alternatives, prescription order waiting time are expressed as functions of the number of pharmacists. In Table I the average waiting time Lq is shown for the multiple server queueing model with no priorities. As the number of pharmacists increase, the average time of an order decreases rapidly.

Table 1 : Expected Waiting Time For Multiple Service Queueing Model With No Priorities.

Number of Pharmacists	Order Waiting Time (in Minutes)
2	11.69
3	1.11
4	0.22
5	0.045

Table 2 represents the priority discipline queueing model. The average waiting time of highest priority order is L_{q1} while that regular orders with the lowest priority is $L_{q2.}$ Again, these waiting times decreases as the number of pharmacists is increased. It can be observed that for the same number of pharmacists is Table 1, the introduction of priorities will decrease the waiting time of the high priority orders while increasing the waiting time of the low priority regular orders.

Table 2: Expected Waiting Time For Priority Discipline Queueing Model

Number of Pharmacists	Waiting Time (in Minutes)	
	highest priority orders	Regular orders
2	2.3	14.89
3	0.56	1.29
4	0.14	0.24
5	0.053	0.045

Table 3 represents the waiting times for the two priorities with the introduction of pre-emptive service. As compared Table 2, where there was no pre-emptive service, the waiting time of the high priority orders are further reduced and that of the low priority regular orders increased. It is possible to essentially eliminate average waiting time for high priority orders when the number of pharmacists is four or more.

Table 3 : Expected Waiting Time For Priority Discipline Queueing Model With Pre-Emptive Service.

Number of Pharmacists	Waiting Time (in Minutes)	
	High Priority orders	Regular orders
2	0.19	15.13
3	0.1018	1.29
4	0.0012	0.024
5	0.0009	0.049

References

1. Bicket WJ and Gangon JP: *Purchasing and Inventory Control for Hospital Pharmacies.* Topics in Hospital Pharmacy Managment 7. 1997.

2. Lehnay B and Paul RJ: *The, Use of Soft Systems Methodology in the Development of a Simulation of out Patient services at Watford General Hospital.* Journal of Operational Research Society 47, 2000.

5. Mehrez A et al.: *On the Implementation of Quantitative Facility Location Models The Case of a Hospital in a Rural Region.* Journal of, Operational Research Society 47, 2000.

6. Sharma JK and Ravindra Babu G : *Operations Research in Health Care-An Up Dated Bibliograhy.* Acta Ciencia Indica, 1998.

7. Woller TW : *Extending the Role of Pharmacy Technicians*. Topics in Hospital Pharmacy Management, 1998.